Radiation Convection in a Thermally Developing Duct Flow of Noncircular Cross Section

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In this paper the interaction of thermal radiation and laminar forced convection in the thermal entrance region of noncircular ducts is numerically investigated. In particular, because of their complex geometry, the paper investigates in detail the combined modes of heat transfer in right triangular and semicircular ducts. The velocity in these geometries is considered fully developed and the temperature of an absorbing-emitting gas is developing in an isothermal duct with nonblack walls. The method of moments is used to describe the radiation contribution that circumvents the partial intergro-differential equation typical of this kind of problem. Also, the methodology using the method of lines (MOL) involving differential and difference formulations and coarse grids is applied to these ducts of noncircular cross section. The results based on 36 lines or less are computed numerically using a Runge-Kutta subroutine. They are presented in terms of axial bulk temperature and mean Nusselt number and are plotted as a function of the optical thickness, wall emissivity, and radiation-conduction parameter. The results compare very well with those for pure convection available in the literature. In addition, it is observed that radiation influences the thermal development of noncircular ducts in a complicated way.

Nomenclature

A = cross-sectional area, m^2
b = characteristic length, m
c_p = specific heat, J-kg^{-1}-K^{-1}
D_h = hydraulic diameter of duct, m
G = total irradiation, W-m^{-2}
G^* = dimensionless value of G, Eq. (5)
K_a = total volumetric absorption coefficient, m^{-1}
k = thermal conductivity, W-m^{-1}-K^{-1}
L = length of duct, m
$\dot{m}$ = mass flow rate, kg-s^{-1}
N = radiation-conduction parameter, Eq. (5)
Nu_T = total Nusselt number, Eq. (16)
Pe = Peclet number, $RePr$
Pr = Prandtl number, ν/α
Q_{ideal} = ideal heat transfer, Eq. (18), W
Q_T = total heat transfer, Eq. (18), W
q_R = thermal radiation heat flux, Eq. (1), W-m^{-2}
q_w = wall heat flux, W-m^{-2}
Re = Reynolds number, $(u_m D_h/\nu)$
T = absolute temperature, K
u, U = dimensional and dimensionless velocity, Eq. (5)
u_m = mean velocity, m-s^{-1}
x, X = dimensional and dimensionless axial coordinates, Eq. (5)
y, Y = dimensional and dimensionless transversal coordinates, Eq. (5)
z, Z = dimensional and dimensionless transversal coordinated, Eq. (5)
α^* = angle in triangular duct, m^2-s^{-1}
α = thermal diffusivity, m^2-s^{-1}
β = extinction coefficient, m^{-1}
ϵ_w = wall emissivity
ν = kinematic viscosity, m^2-s^{-1}
ρ = density, kg-m^{-3}
σ = Stefan-Boltzmann constant, W-m^{-2}-K^{-4}
τ_b = optical thickness, Eq. (5)
ϕ = dimensionless temperature, Eq. (5)
Ω = heat transfer efficiency, Eq. (18)

Subscripts
b = mean bulk
C = conduction
e = entrance
R = radiation
ref = reference
T = total
w = wall (circumferential)

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Introduction

IT is a well-known fact that, at a high temperature, thermal radiation has a pronounced effect on the heat transfer characteristics of a participating gas in a duct flow. For such flows it has been proven that the conventional Nusselt number is no longer valid and that deviation increases as the magnitude of the radiation parameters increases. Additionally, this information is of paramount interest in the design of compact heat exchangers, turbines, etc., being used in nuclear power plants and other energy-related industries.

The study deals with an analytical/numerical solution of combined convection and radiation in the thermal entry region of right triangular and semicircular ducts subjected to a uniform wall temperature (see Figs. 1 and 2). In fact, literature surveys by Eckert et al.,[1,2] Martynenko,[3] Soloukhin and Martynenko,[4] Shah and Bhatti,[5] and Shah and London[6] indicate that significant work has been published in the area of forced convection in ducts on noncircular geometry. However, the investigations cited in these reviews, along with the work of Wibulswas,[7] Chandrupatta and Sastri,[8] Haji-Sheikh et al.,[9] Lakshminarayanan and Haji-Sheikh,[10] Normua and Haji-Sheikh,[11] Sparrow and Haji-Sheikh,[12,13] Hong and Bergles,[14] and Manglik and Bergles,[15,16] have been restricted to situations that do not account for the simultaneous effect of forced convection and radiation in the thermal entrance region of ducts where the flowing medium is a participating gas.

Some studies related to the present problem, but restricted to circular pipes under isothermal wall conditions, were carried out by several researchers. Among those, Pearce and Emery[17] and Pearce[18] considered the flow of gas with parabolic velocity distributions and solved the descriptive conservation equations employing a numerical scheme. Echigo et al.[19] analyzed the gas flow as a conjugate problem in the fluid domain, using finite-difference techniques. These authors ex-

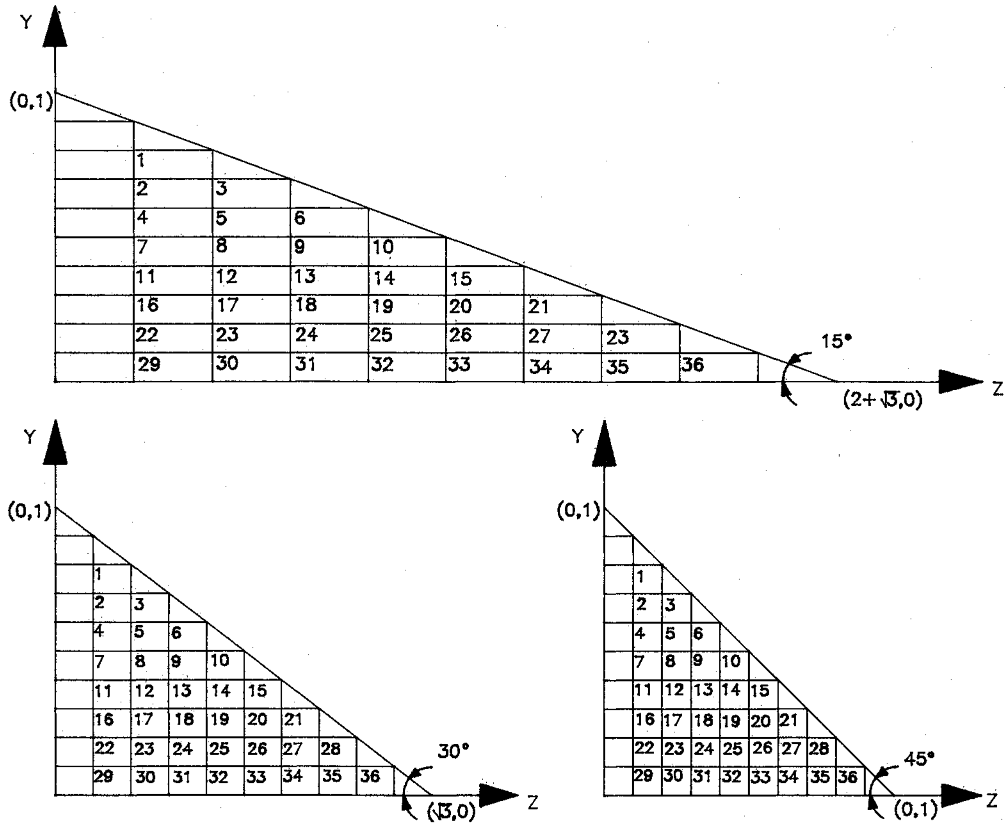

Fig. 1 Coordinate system for coarse grid for right triangular duct.

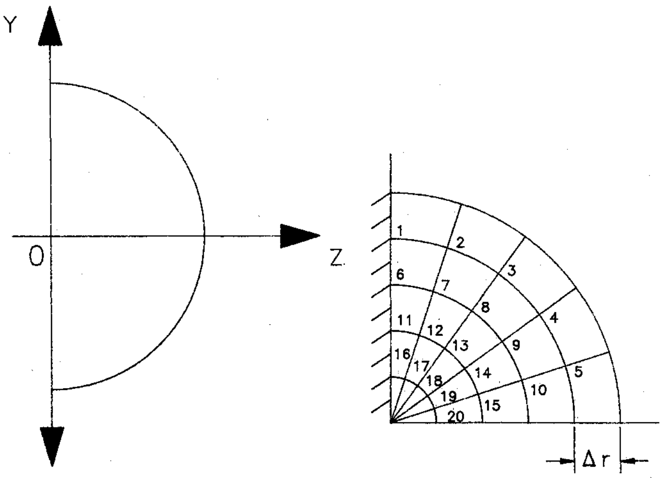

Fig. 2 Coordinate system for coarse grid for semicircular duct.

amined the energy equation with two-dimensional radiative heat transfer allowing a propagation upstream from the entrance with two-dimensional radiative heat transfer allowing a propagation upstream from the entrance of the heating section. It was necessary in Pearce and Emery,[17] Pearce,[18] and Echigo et al.[19] to evolve an alternative solution procedure in which the integral term and the differential terms of the energy equation are solved consecutively and the calculation procedure is repeated until convergence is achieved. Huang and Lin[20] presented a numerical analysis of combined laminar forced convection and thermal radiation in the thermally developing flow of circular cross section. They also considered both axial conduction and radiation. Included in their study is an analysis of absorbing, emitting, and isotropically scattering gray fluid bounded by a hot wall having a sudden change in temperature. They solved the governing equation numerically by the successive over-relaxation finite-difference method with an iterative procedure. Thermal radiation is considered by solving the exact integral equations for the source function and boundary intensity using the nodal approximation technique. In Yener and Fong[21] solution to the energy equation is developed in the form of a Fourier series, and the radiation part is analyzed by the F_N method.

Turning attention to the modeling of radiation in a participating medium, it has been demonstrated that its rigorous formulation involves the solution of a nonlinear integral equation. However, various approximate and accurate differential methods, for example, the method of moments as described by Özisik,[22] model radiation transfer by an elliptic partial differential equation defining the irradiation distribution in the medium. Ratzel and Howell[23] found that their predicted results, using the method of moments, deviate by almost 20% from the real case for an optical thickness of 1 and black wall conditions. Also, it has a tendency to break down near the boundaries. However, for the gray surface ($\epsilon = 0.5$, the common value for metal ducts), the error decreased to less than 10%. This error will further be reduced as the optical thickness is increased. Campo and Schuler[24] adopted the method of moments to model thermal radiation in the developing flow in pipes of circular cross section. Their results are in good agreement with those of Pearce and Emery[17] and Echigo et al.[19]

In this paper the developing temperature is determined by solving the three-dimensional energy equation accounting for a temperature-dependent source term via the method of lines (cf. Liskovets[25]). According to this method, the transversal derivatives in this parabolic partial differential equation are replaced by finite-difference formulations, while the axial derivatives remain continuous. Correspondingly, the integration domain is divided into a collection of lines parallel to the axial coordinate of the duct. Thus, the partial differential energy equation is being replaced by a system of nonlinear ordinary differential equations of first order, where the dependent variable is the temperature along each line, and the independent variable is the axial coordinate. It should be added that, for computational purposes, the retention of equal transversal intervals in the presence of irregular boundaries constitutes a complicating feature requiring special equations for nodes in their neighborhood (cf. Carnahan et al.[26]). To prevent this obstacle, the construction of the grid in the duct cross section is such that the dividing lines in the grid itself coincide with the irregular boundaries.

In this paper the resulting coupled system of nonlinear ordinary differential equations and algebraic equations, based on relatively coarse grids, has been solved numerically by an adaptation of a Runge-Kutta algorithm. The results have been presented in terms of the distribution of the mean bulk temperature and the total Nusselt number.

Formulation

Consider a thermally developing, steady laminar flow of a radiating fluid in a noncircular duct. The fully developed laminar gas flow enters the heated section of the duct at the origin of the axial coordinate x, with a temperature of T_e, while the surface of the duct is maintained at an isothermal temperature T_w for $x>0$. Furthermore, the participating gas in the region of thermal development is assumed to be gray, emitting, and absorbing. In addition, the physical properties of the fluid are considered constant. Therefore, the energy equation, in the absence of viscous dissipation, axial heat conduction, and axial radiation, can be written in dimensionless forms as follows:

$$u\frac{\partial T}{\partial x}=\alpha\left(\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)-\frac{1}{\rho c_p}\operatorname{div} q_R \tag{1}$$

where $u = u(y,z)$ designates the velocity profile determined from the equation

$$\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=\frac{1}{\mu}\frac{\mathrm{d}p}{\mathrm{d}x} \tag{2}$$

Also, in Eq. (1), x, y, and z are the axial and transversal coordinates of the ducts, respectively, and T is the temperature sought.

The term div q_R in the second term of the right side of Eq. (1) corresponds to participating thermal radiation and is retained here in recognition of its importance in high-temperature gas flows. This contribution is modeled here by an approximate differential method, such as the method of moments in two-dimensional form (cf. Özisik[22]). Accordingly, the radiative transfer equation (RTE) is conveniently expressed in differential form as follows:

$$\operatorname{div} q_R = -K_a(G-4\sigma T^4) \tag{3}$$

where G is the irradiation given by the equation

$$\frac{\partial^2 G}{\partial x^2}+\frac{\partial^2 G}{\partial y^2}+\frac{\partial^2 G}{\partial z^2}=3\beta K_a(G-4\sigma T^4) \tag{4}$$

In Eq. (4) β represents the extinction coefficient.

Adopting the following dimensionless variables and parameters as

$$U=\frac{u}{u_m},\qquad \phi=\frac{T}{T_w}$$

$$Y=\frac{y}{b},\qquad Z=\frac{z}{b}$$

$$X=\frac{x}{D_h Pe},\qquad Re=\frac{u_m D_h}{\nu},\qquad Pe=RePr \tag{5}$$

$$N=\frac{4\sigma T_w^3}{kK_a},\qquad \tau_b=K_{ab},\qquad G^*=\frac{G}{4\sigma T_w^4}$$

where $T_{\mathrm{ref}}=T_w$ is defined as a reference temperature, and b represents the characteristic length of the duct.

Based on the previously given dimensionless parameters, the energy equation [Eq. (1)] and the equation of radiative transfer [Eq. (4)] lead to a new system of partial differential equations described by

$$U\frac{\partial \phi}{\partial X}=\frac{\partial^2\phi}{\partial Y^2}+\frac{\partial^2\phi}{\partial Z^2}+N\tau_b^2(G^*-\phi^4) \tag{6}$$

and

$$\frac{1}{Pe^2}\frac{\partial^2 G^*}{\partial X^2}+\frac{\partial^2 G^*}{\partial Y^2}+\frac{\partial^2 G^*}{\partial Z^2}=3\tau_b^2(G^*-\phi^4) \tag{7}$$

Inspection of Eq. (7) reveals that, in general, for gas flows in small to moderate diameter pipes, $Pe \gg 1$. In fact, Echigo et al.[19] conducted a series of numerical experiments for a conjugate version of this problem related to circular pipes. Their results showed that the axial radiation penetrated one or two diameters in the upstream region of the heat-exchange section. This conclusion was also verified by Pearce,[18] who used a more formal order-of-magnitude analysis for circular pipes. Echigo et al.[19] proposed a radiative Pe that serves to define the threshold of the axial radiation effects. That is, in terms of the hydraulic diameter,

$$Re_{D_h}Pr\left(\frac{\tau_b}{N}\right)>10 \tag{8}$$

Therefore, applying the preceding criteria to Eq. (7) indicates that the axial variation of G^* may be dropped, which makes the outcome a partial differential equation:

$$\frac{\partial^2 G^*}{\partial Y^2}+\frac{\partial^2 G^*}{\partial Z^2}=3\tau_b^2(G^*-\phi^4) \tag{9}$$

Now, the related boundary conditions for Eqs. (6) and (9) are expressed in dimensionless form as follows.

At the inlet of the duct,

$$\phi=\phi_e,\qquad X=0 \tag{10}$$

On the wall of the duct,

$$\phi=1,\qquad X>1 \tag{11}$$

$$\frac{\partial G^*}{\partial n}=-\frac{3}{2}\lambda_w\tau_b(G_w^*-1),\qquad X>1 \tag{12}$$

where n indicates the normal direction to the wall, and

$$\lambda_w=\frac{\epsilon_w}{2-\epsilon_w} \tag{13}$$

Thermal Quantities of Interest

The physical quantities of interest for practical applications of internal forced convection and radiation exposed to isothermal wall conditions are the dimensionless bulk temperature $\phi_b(x)$,

$$\phi_b(X)=\frac{\int_A U\phi\,\mathrm{d}A}{\int_A U\,\mathrm{d}A} \tag{14}$$

and the local surface heat flux q_w,

$$q_{wT}=q_{wC}+q_{wR} \tag{15}$$

where q_{wC} and q_{wR} represent local conductive and local radiative transfer at the duct wall, respectively. These quantities lead to the definition of the total Nusselt number:

$$Nu_T=\frac{q_w D_h}{K(T_w-T_b)} \tag{16}$$

Next, combining Eqs. (12), (13), (15), and (16), one can write

$$Nu_T = \frac{\left[2\frac{\partial \phi}{\partial n}\,\Big|_w - 4N\lambda_w (G_w^* - 1) \right]}{1 - \phi_b} \tag{17}$$

In this equation the first term of the right side corresponds to local conductive heat transfer and has been calculated using a finite-difference formulation for the temperature gradient. The second term of the right side described the local radiative heat transfer and has been evaluated by using the method of moments (cf. Özisik[22]).

At this stage, it should be stressed that the calculation of the total heat transfer involving the total Nusselt number is more elaborate, with the sole purpose of comparing our results against the asymptotic solutions that other investigations have published in the open literature. However, the total rate of heat transfer in a duct of length L may be easily determined from an energy balance between two axial stations $x = 0$ and $x = L$. In fact, it relates the bulk temperature ratios, ϕ_b/ϕ_{ref}, to the so-called heat transfer efficiency Ω, defined as

$$\Omega = Q_T / Q_{\text{ideal}} \tag{18}$$

where Q_T and Q_{ideal} represent the total and ideal heat transfer rates, respectively. Therefore, the heat transfer efficiency is related to the appropriate local temperatures as follows:

$$\Omega = \frac{\phi_e - \phi_{bL}}{\phi_e - 1} \tag{19}$$

where $\phi_{bL} = \phi_b(L)$.

Method of Solution

A numerical method is necessary to solve the nonlinear problem under consideration. Accordingly, the system of Eqs. (6) and (9), subject to the boundary conditions expressed by Eqs. (10–13), was numerically solved by combining the method of lines (MOL) and the Runge-Kutta algorithm.

The temperature distribution $\phi(Y,Z)$ has been determined by the hybrid MOL using uniform grids. The MOL has been described by Liskovets[25] as a solution technique that transforms a partial differential equation (PDE) into an appropriate system of ordinary differential equations for a parabolic PDE involving three independent variables as in Eq. (6). The region of integration may be divided into lines parallel to the axial coordinate x. Accordingly, the axial derivative, $\partial\phi/\partial x$, remains continuous, whereas the transversal derivatives, $\partial^2\phi/\partial Y^2$ and $\partial^2\phi/\partial Z^2$, are replaced by finite-difference formulations using values of unknown quantities on the same line and on neighboring lines. Hence, this simple mathematical concept generates a system of ordinary differential equations of first order where the dependent variables are described along each line in terms of a single independent variable, X. It may therefore be readily solved numerically employing a standard Runge-Kutta algorithm. Using the finite-difference method, the partial differential equation for G^*, Eq. (9) can be discretized into an associated system of algebraic equations. Thus, this system is carried out at each axial station, X, using an adaptation of the Gaussian elimination algorithm for the numerical determination of G^* at each cross section.

Table 1 Nusselt number results for laminar forced convection ($N = 0$)

	Right triangular duct					
	15 deg		30 deg		45 deg	
	$Nu_{m,T}$					
$1/X$	Present Study	Ref. 10	Present Study	Ref. 10	Present Study	Ref. 10
10	2.54	2.60	2.66	2.65	2.61	2.60
20	3.02	3.10	3.04	3.02	2.87	2.83
30	3.41	—	3.36	—	3.11	—
40	3.73	—	3.62	—	3.33	—
60	4.27	—	4.07	—	3.70	—
80	4.70	—	4.42	—	4.00	—
100	5.13	5.21	4.77	4.82	4.30	4.21
120	5.39	—	4.97	—	4.48	—
160	6.10	—	5.54	—	4.97	—
180	6.30	—	5.69	—	5.09	—
200	6.64	6.59	6.02	6.09	5.32	5.28
1000	13.67	12.08	11.20	10.63	9.08	9.17
2000	16.72	16.27	14.21	13.80	11.39	11.78
10,000	36.33	33.72	29.57	25.29	23.34	20.69

Results and Discussion

The problem considered in this analysis contains a numerical scheme for solving the simultaneous effect of convection and radiation in a duct of noncircular cross section, mainly, the right triangular duct of 15, 30, and 45 deg and semicircular ducts. These ducts are also subjected to constant wall temperature. Numerical solutions were generated for these geometries. Because of their complicated cross section, all computations were performed in double precision on the VAX 8800. In addition, this analysis also contains numerous physical parameters, such as τ_b, N, ϵ, X, ϕ_b and Nu_T. The ϕ_b, Eq. (14), and the Nu_T, Eq. (17), have been calculated numerically in the thermal entrance region of right triangular and semicircular ducts using the MOL. These have been presented graphically in Figs. 4–15.

In order to prove the validity of the proposed methodology, numerical solutions for a laminar thermal entrance region absent of radiation ($N = 0$) have been carried out. The values of Nu_m from these solutions are listed in Tables 1 and 2 along

Table 2 Nusselt number results for laminar forced convection ($N = 0$)

	Semicircular duct	
	$Nu_{m,T}$	
$1/X$	Manglik and Bergles[16]	Present study
10.27	6.34	6.827
18.73	7.07	7.701
28.01	7.75	8.526
34.25	8.31	9.02
51.28	8.79	10.22
77.52	9.92	11.71
105.26	10.60	12.59
144.93	12.40	14.70
222.22	15.40	17.39

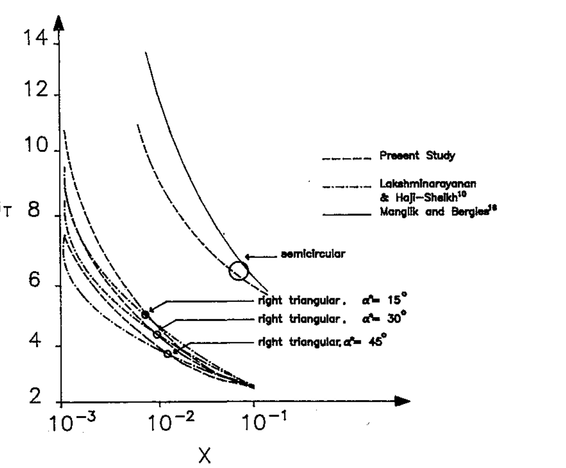

Fig. 3 Comparison of the total Nusselt number for laminar forced convection ($N = 0$).

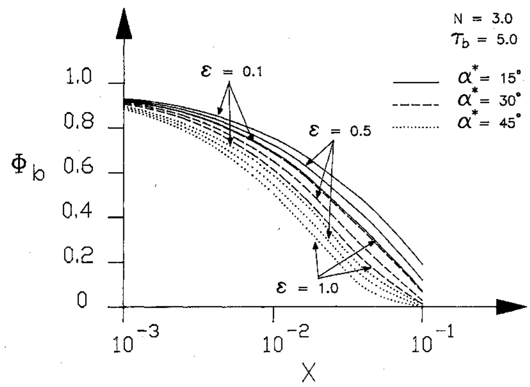

Fig. 4 Effects of wall emissivity on the mean bulk temperature for right triangular ducts.

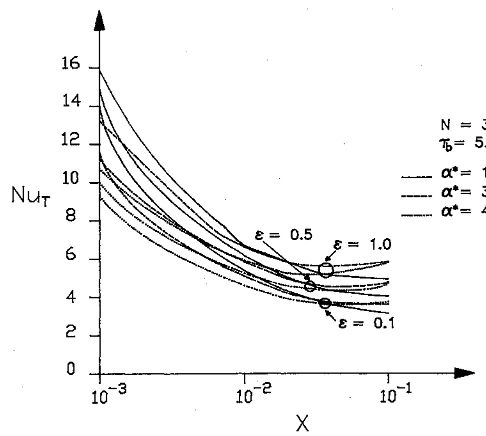

Fig. 5 Effects of wall emissivity on the total Nusselt number for right triangular ducts.

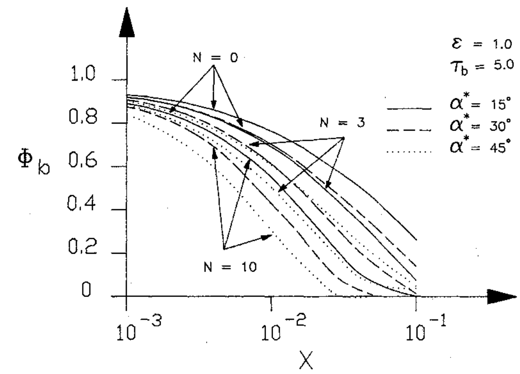

Fig. 6 Effects of radiation-conduction parameter on the mean bulk temperature for right triangular ducts.

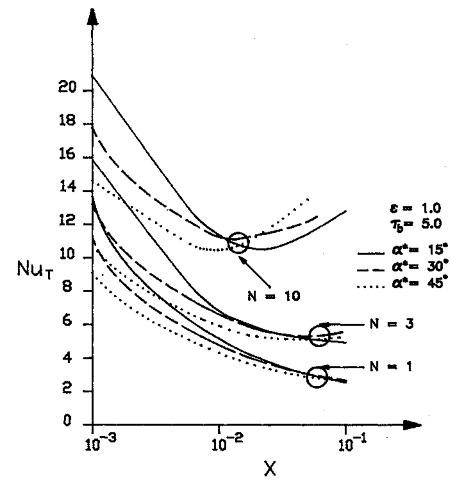

Fig. 7 Effects of radiation-conduction parameter on the total Nusselt number for right triangular ducts.

with benchmark solutions of Lakshminarayanan and Haji-Sheik[10] for the right triangular duct and Manglik and Bergles[15,16] for the semicircular duct. Tables 1 and 2 indicate a very close agreement with the previously mentioned references. This agreement further supports the validity of the proposed methodology. The data in this table were used to draw the curves in Fig. 3.

Turning attention to the combined forced convection and radiation, $\phi = 0.5$ (the entrance-to-wall temperature ratio) is used throughout the calculation. Furthermore, in order to increase the accuracy of the results presented, cubic spline interpolation has been applied to integrate the velocity and the mean bulk temperature. This has been accomplished by using the two-dimensional tensor product BS21N and BS21G in the IMSL Library Package.[27] The following section is devoted to the discussion of each geometry.

Right Triangular Duct

The calculation for fluid flow and heat transfer is performed for right triangular ducts with 15-, 30-, and 45-deg angles, as shown in Fig. 1. The former study by Yang et al.[28] showed that CPU time is nearly proportional to the number of lines, and satisfactory accuracy can be reached by using a relative coarse grid. Therefore, a system having 36 lines is used in all cases. The CPU time for solving this problem is 120 s on a VAX 8800 computer. Figures 4 and 5 represent the axial variation of the mean bulk temperature and total Nusselt number with a radiation-conduction parameter of $N = 3$ and optical thickness of $\tau_b = 5$ for emissivities of $\epsilon = 0.1$, 0.5, and 1.0. Figure 4 indicates that the emissivity has a minimal effect on the mean bulk temperature of the gas. The fluid temperature in this region is governed by convection and radiation transfer in the medium and is weakly affected by wall radiation. It is further observed that the bulk temperature increases with decreases in both wall emissivity and the angle of the right triangular duct. Figure 5 also indicates that the Nusselt number is largest at the entrance region of the right triangular duct with a 45-deg angle. Then it gradually decreases until it reaches a minimum value. Eventually, it tends to increase without approaching an asymptotic value. The same behavior has been observed by other investigators[17,19,20,22] for circular ducts. The location of the minimum value shifts to the right for a 30-deg angle duct. However, for right triangular ducts with a 15-deg angle, no obvious *Nu* recovery is observed in this evaluated region. It is believed that this is due mainly to the effect of the sharp corner of the duct.

Figures 6 and 7 represent the axial variation of the mean bulk temperature and the total Nusselt number with $\epsilon = 1.0$ and $\tau_b = 5.0$ for conduction-radiation parameters of $N = 0$, 3, and 10. In Fig. 6 conduction radiation represents the relative importance of conduction compared to radiation at the wall. It is seen from this figure that, by increasing N, the mixed mean temperature of the medium levels more toward the inlet of the channel. In short, it can be deduced that wall conduction tends to promote the state of thermal uniformity in the medium. Figure 7 illustrates that, by increasing N, the position of the minimum convective Nusselt number shifts more toward the inlet of the channel, indicating a faster thermal development in the fluid.

The axial variations of the mean bulk temperature and Nusselt numbers with $N = 3$ and $\epsilon = 1.0$ for $\tau_b = 0.5$, 1.0, and 5.0 are shown in Figs. 8 and 9. Figure 8 indicates that the effect of optical thickness on the fluid and wall temperature profiles is found to be of considerable significance. It is also noted that, as a result of the heat transfer augmentation effect of radiation, the mixed mean temperature of the medium is highest at the entrance of the duct, and it is possible to level off near the end of the channel. This is the indication that the gas actively participates in the radiative transfer by absorbing the energy radiated from the walls, which is then redistributed throughout the gas and back to the walls. On the other hand, as the optical thickness increases, the temperature of the fluid rises very rapidly along the channel. Figure 9 indicates that the variation in optical thickness has a great impact on convective heat transfer. At the low values of optical thickness, the total Nusselt number decreases monotonically in the axial direction, but unlike pure convection for which an asymptotic value is reached. In this case the total Nusselt number decreases at an even greater rate as the end of the channel is approached.

Semicircular Duct

For this configuration the calculations for the fluid flow and heat transfer are performed based on 20 lines, as shown in Fig. 2.

Figures 10 and 11 represent the variation of the bulk temperature and total Nusselt number with $N=3.0$ and $\tau_b=5.0$ for $\epsilon=0.1$, 0.5, and 1.0. Figure 10 illustrates that wall emissivity has a minimal effect on the mixed mean bulk temperature. The same behavior with the same conditions was observed in Fig. 4 for the case of the right triangular duct; however, a smaller variation is observed here. Figure 11 indicates that the total Nusselt number increases with increasing wall emissivity. Furthermore, it is observed that at $\epsilon=1.0$ the $Nu_{m,T}$ number decreases gradually at the entrance region of the semicircular duct until it reaches a minimum number and dramatically increases at that point.

Figures 12 and 13 depict the axial variation of the mean bulk temperature and total Nusselt number with $\epsilon=1.0$ and $\tau_b=5.0$ for $N=0$, 1, and 3. Figure 12 indicates that the mixed mean bulk temperature increases very rapidly with the increase of N. The curve $N=0$, in this figure, relates to pure forced convection. Figure 13 represents the same behavior as was

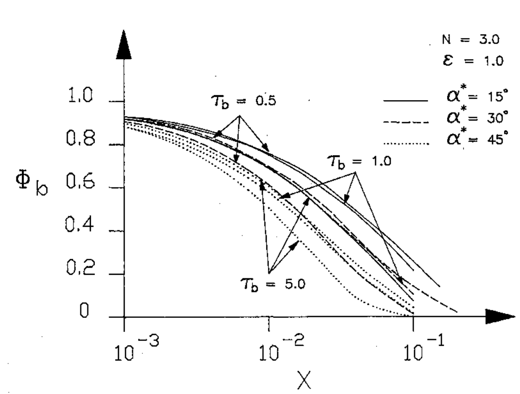

Fig. 8 Effect of optical thickness on the mean bulk temperature for right triangular ducts.

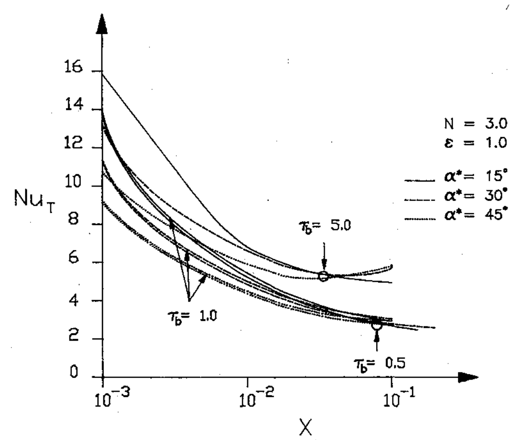

Fig. 9 Effect of optical thickness on the total Nusselt number for right triangular ducts.

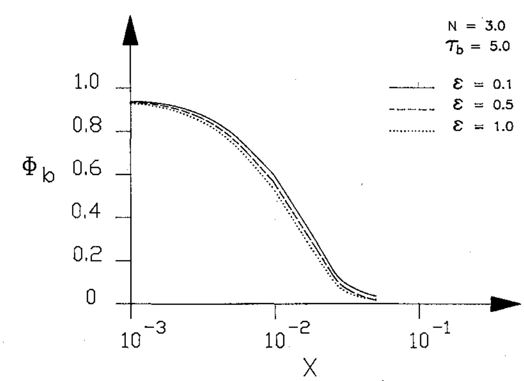

Fig. 10 Effect of wall emissivity on the mean bulk temperature for a semicircular duct.

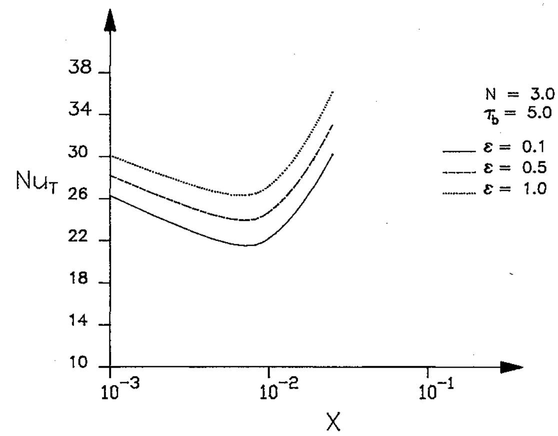

Fig. 11 Effect of wall emissivity on the total Nusselt number for a semicircular duct.

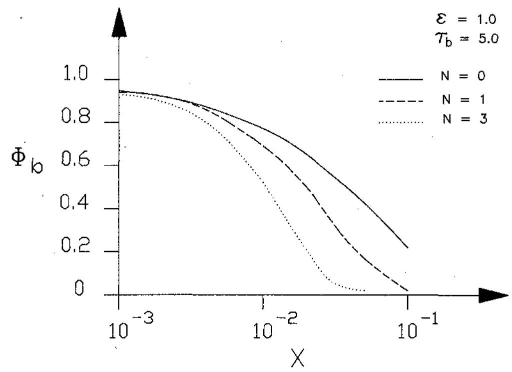

Fig. 12 Effect of radiation-conduction parameter on the mean bulk temperature for a semicircular duct.

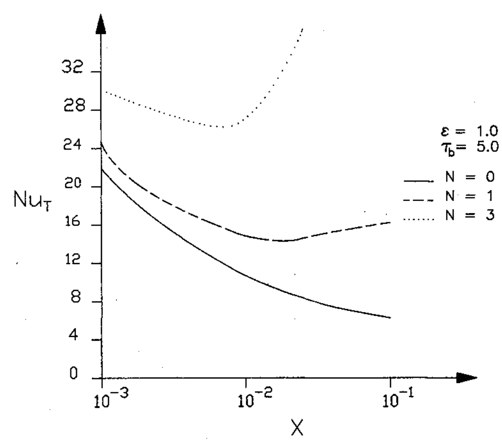

Fig. 13 Effect of radiation-conduction parameter on the total Nusselt number for a semicircular duct.

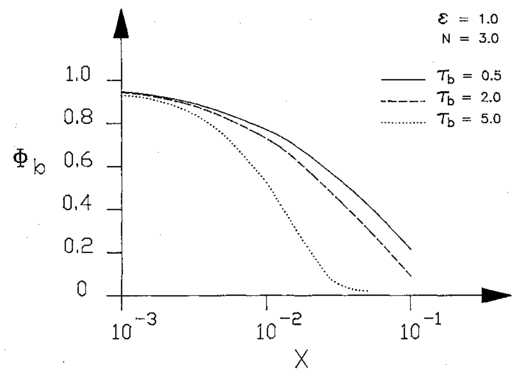

Fig. 14 Effect of optical thickness on the mean bulk temperature for a semicircular duct.

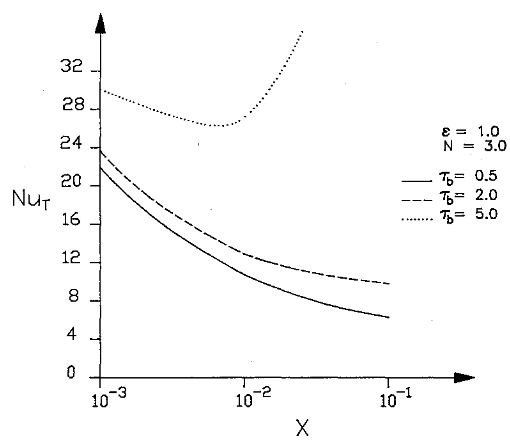

Fig. 15 Effect of optical thickness on the total Nusselt number for a semicircular duct.

discussed earlier. However, it can be observed that an increase in the conduction-radiation parameter will significantly enhance the total Nusselt number.

Figures 14 and 15 represent the variation of the axial mean bulk temperature and the total Nusselt number with $\epsilon = 1$ and $N = 3$ for $\tau_b = 0.5$, 2.0, and 5.0. Figure 14 indicates that the optical thickness has a pronounced effect on the mixed mean bulk temperature. At the entrance of the semicircular duct, there is basically no deviation in the bulk temperature at different optical thickness. This can be explained by the fact that the forced convection is dominant at that section. However, as the gas passes through the duct, a larger deviation is observed in the bulk temperature with increasing optical thickness. Obviously this is due to the facts that, at the end of the channel, the dominant term is radiative. Figure 15 depicts the total Nusselt number for the same conditions, and the same behavior can be observed in this case, as the optical thickness enhances the total Nusselt number. In addition to this, the variation of the $Nu_{m,T}$ number in the presence of strong radiation shows that a state of fully thermal development is never reached.

Concluding Remarks

In this paper the combination of forced convection and thermal radiation has been examined for the flow of an absorbing-emitting gas in isothermal ducts of right triangular and semicircular cross section. The significant role played by radiation in the thermal entry region has been successfully represented by the method of moments. An explicit finite-difference for laminar gas flows that takes advantage of the MOL has been adopted in the thermal entrance region of these ducts. The nodes have been placed carefully along the surfaces to avoid the usual difficulties associated with the irregular boundaries. The use of coarse grids in the cross section of the ducts yields a relatively small system of first-order ordinary differential equations. Hence, this system can be readily solved by either an analytical or a numerical technique. The numerical scheme was found to be simple, accurate, and efficient. It is ideally suited for a combined mode of heat transfer. A comparison of these results for pure convection ($N = 0$) agrees well with previous investigations. The effects of the different physical parameters were systematically studies. From the cases studied the following conclusions were obtained:

1) Interaction of radiation with convection increases the convective heat flux at the wall and, in general, results in an augmentation of the heat transfer process.

2) The effect of wall emissivity on the mixed mean bulk temperature in the medium for these geometries is minimal.

3) At an even moderate optical thickness, the behavior of the convective Nusselt number is substantially altered.

4) An increase in the conduction-radiation parameter produces more uniform temperatures in the wall. It also promotes thermal development of the fluid by producing more uniform temperatures in the medium.

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